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LETTER TO THE EDITOR

The quantum q-deformed symmetric top: an exactly solved model

Zhe Chang

CCAST (World Laboratory), PO Box 8730, Beijing 100080, People's Republic of China, and (mailing address) Institute of Theoretical Physics, Academica Sinica, PO Box 2735, Beijing 100080, People's Republic of China

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Abstract. By means of symplectic geometry, we discuss the quantum q-deformed symmetric top system. Quantum group $SU_q(2)$ is realized in the system. We show that the quantum q-deformed symmetric top system in which quantum group is realized and the quantum undeformed symmetric top system in which Lie group is realized correspond to the same Heisenberg equations and then the quantum q-deformed symmetric top is an exactly solved model.

It is well known that the symmetric top model plays an important role in some physical systems such as molecules and nuclei [1, 2]. The rotational motions in these systems can be described by the symmetric top model. However, fine structures of experimental data show that nonlinear modifications must be introduced and the Lie group symmetry possessed by the whole is then violated. So it is necessary to investigate a deformed symmetric top model which can give the nonlinear modifications and possesses some symmetries. In [3], we showed that as long as a suitable Hamiltonian and symplectic structure were chosen, a quantum group [4–13] can be realized in the symmetric top system. In this paper, we show that the quantum q-deformed symmetric top model and the quantum undeformed symmetric top model correspond to the same Heisenberg equations and then the quantum q-deformed symmetric top system is an exactly solved model.

The standard Hamiltonian of a symmetric top is of the form

$$H=\frac{J_1^2+J_2^2}{2I}+\frac{J_3^2}{2I_3}. \tag{1}$$

We know that the generalized phase space of the top system can be of the form

$$M_0:\quad J_1^2+J_2^2+J_3^2=J_0^2. \tag{2}$$

On the phase space M_0, we define a symplectic form as

$$\omega_0=\frac{1}{2J_0^2}\sum_{ijk}^{3}\varepsilon_{ijk}J_i\,\mathrm{d}J\!\!\!B_j\wedge\mathrm{d}J_k. \tag{3}$$

Let V_+ and V_- be two open sets in M_0 defined by

$$V_\pm=\{x\in M_0|J_0\pm J_3(x)\neq 0\}. \tag{4}$$

We denote by w_+ and w_- the complex functions on V_+ and V_-, respectively, defined by

$$w_\pm = \frac{J_1 \mp iJ_2}{J_0 \pm J_3}. \tag{5}$$

In $V_+ \cap V_-$ we have

$$w_+ w_- = 1. \tag{6}$$

From (5), we can obtain

$$\begin{aligned} J_1 &= J_0 \frac{w_\pm + \bar{w}_\pm}{1 + w_\pm \bar{w}_\pm} \\ J_2 &= \pm \mathrm{i} J_0 \frac{w_\pm - \bar{w}_\pm}{1 + w_\pm \bar{w}_\pm} \\ J_3 &= \pm J_0 \frac{1 - w_\pm \bar{w}_\pm}{1 + w_\pm \bar{w}_\pm}. \end{aligned} \tag{7}$$

This enables us to rewrite ω_0 as

$$\omega_0 | V_\pm = 2\mathrm{i} J_0 \frac{\mathrm{d}\bar{w}_\pm \wedge \mathrm{d}w_\pm}{(1 + w_\pm \bar{w}_\pm)^2}. \tag{8}$$

The restrictions of ω_0 to the open sets V_+ and V_- are exact

$$\omega_0 | V_\pm = \mathrm{d}\theta_\pm \tag{9}$$

where

$$\theta_\pm = 2\mathrm{i} J_0 \frac{\bar{w}_\pm \, \mathrm{d}w_\pm}{1 + w_\pm \bar{w}_\pm}. \tag{10}$$

The quantum line bundle L_q of the symplectic manifold (M_0, ω_0) exists if and only if the de Rham cohomology class $[-h^{-1}\omega_0]$ of $-h^{-1}\omega_0$ is integral. Integrating the form on the right-hand side of (3) over the sphere J^2, we obtain $4\pi J_0 h^{-1}$, which must be an integer if $[-h^{-1}\omega_0]$ is integral. Hence we have

$$J_0 = j\hbar \qquad j = 0, \tfrac{1}{2}, 1, \ldots \tag{11}$$

where the integer $2j \in Z$ is the Chern class of the bundle L_q.

Used (7) and (8), the Hamiltonian vector fields X_{J_i} can be expressed as follows

$$\begin{aligned} X_{J_1} | V_\pm &= \frac{\mathrm{i}}{2} \left[(w_\pm^2 - 1) \frac{\partial}{\partial w_\pm} - (\bar{w}_\pm^2 - 1) \frac{\partial}{\partial \bar{w}_\pm} \right] \\ X_{J_2} | V_\pm &= \mp \frac{1}{2} \left[(w_\pm^2 + 1) \frac{\partial}{\partial w_\pm} + (\bar{w}_\pm^2 + 1) \frac{\partial}{\partial \bar{w}_\pm} \right] \\ X_{J_3} | V_\pm &= \mp \mathrm{i} \left[\bar{w}_\pm \frac{\partial}{\partial \bar{w}_\pm} - w_\pm \frac{\partial}{\partial w_\pm} \right]. \end{aligned} \tag{12}$$

These equations, together with (10), yield

$$\hat{J}_1 = \frac{1}{2}\hbar(w_\pm^2 - 1)\frac{\partial}{\partial w_\pm} - J_0 w_\pm$$

$$\hat{J}_2 = \pm\frac{i}{2}\hbar(w_\pm^2 + 1)\frac{\partial}{\partial w_\pm} - iJ_0 w_\pm \tag{13}$$

$$\hat{J}_3 = \mp\left(-\hbar w_\pm \frac{\partial}{\partial w_\pm} + J_0\right).$$

The above operators give the Lie bracket realization of the Lie group SU(2)

$$[J_i, J_j] = -i\hbar\varepsilon_{ijk}J_k \tag{14}$$

or

$$[\hat{J}_3, \hat{J}_\pm] = \mp\hbar\hat{J}_\pm \qquad [\hat{J}_+, \hat{J}_-] = -\hbar 2\hat{J}_3. \tag{15}$$

Using this algebra and the Hamiltonian (1), we can easily write down the Heisenberg equations of the quantum symmetric top system as

$$i\dot{\hat{J}}_1 = [\hat{J}_1, \hat{H}] = \frac{I - I_3}{2II_3}(\hat{J}_2\hat{J}_3 + \hat{J}_3\hat{J}_2)$$

$$i\hbar\dot{\hat{J}}_2 = [\hat{J}_2, \hat{H}] = \frac{I_3 - I}{2II_3}(\hat{J}_1\hat{J}_3 + \hat{J}_3\hat{J}_1) \tag{16}$$

$$i\hbar\dot{\hat{J}}_3 = [\hat{J}_3, \hat{H}] = 0.$$

It is well known that the stationary states of the symmetric top can be given by the Wigner D-functions, D^J_{MK},

$$D^J_{MK}(\alpha, \beta, \gamma) = e^{-iM\alpha - iK\gamma} d^J_{MK}(\beta) \tag{17}$$

where

$$d^J_{MK}(\beta) = [(J+M]!(J-M)!(J+K)!(J-K)!]^{1/2}$$
$$\times \sum_\nu [(-)^\nu (J-M-\nu)!(J+K-\nu)!(\nu+M-K)!\nu!]^{-1} \tag{18}$$
$$\times (\cos\beta/2)^{2J+K-M-2\nu}(-\sin\beta/2)^{M-K-2\nu}$$

(α, β and γ are the Euler angles). They are eigenfunctions of $\hat{J}_3$, $\hat{J}_z$ and $\hat{J}^2$

$$\hat{J}_3 D^J_{MK} = K\hbar D^J_{MK} \qquad \hat{J}_z D^J_{MK} = M\hbar D^J_{MK}$$
$$\hat{J}^2 D^J_{MK} = J(J+1)\hbar^2 D^J_{MK} \tag{19}$$

where $\hat{J}_z$ is the projection of $\hat{J}$ onto the z-axis of the lab-fixed coordinate system. So the eigenvalues of the Hamiltonian of the symmetric top system are

$$E_{JK} = \frac{\hbar^2}{2I}J(J+1) + \frac{\hbar^2 K^2}{2}\left(\frac{1}{I_3} - \frac{1}{I}\right). \tag{20}$$

Now we discuss the symmetric top system with deformed Hamiltonian and deformed symplectic structure. Let us begin with writing down the deformed Hamiltonian of the symmetric top

$$H_q=\frac{I-I_3}{2II_3}J_3^2+\frac{1}{2I}\left(\frac{\gamma}{\sinh\gamma}\hat{J}_+\hat{J}_-+[\hat{J}_3]_q[\hat{J}_3+\hbar]_q\right) \tag{21}$$

where $[x]_q=(q^x-q^{-x})/(q-q^{-1})$ and $\gamma=\log q$.

We know that the generalized phase space of the deformed Hamiltonian system is of the form [3]

$$M_0^q:\quad J_1^2+J_2^2+\frac{(\sinh\gamma J_3)^2}{\gamma\sinh\gamma}=J_q^2 \tag{22}$$

where J_q is a constant. In fact J_q is of the form $J_q=\sinh\gamma J_0/\sqrt{\gamma\sinh\gamma}$. It is obvious that the phase space M_0^q reduces to the usual phase space M_0 when the deformation parameter γ approaches zero.

On the phase space M_0^q, a symplectic form is defined as [14, 15]

$$\omega_q=\frac{1}{J_q^2}\left(J_1\,\mathrm{d}J_2\wedge\mathrm{d}J_3+J_2\,\mathrm{d}J_3\wedge\mathrm{d}J_1+\frac{\tanh\gamma J_3}{\gamma}\mathrm{d}J_1\wedge\mathrm{d}J_2\right). \tag{23}$$

We intrdocue two open sets $U_\pm$ on the phase space M_0^q,

$$U_\pm=\left\{x\in M_0^q\,\middle|\,J_q\pm\frac{\sinh\gamma J_3}{\sqrt{\gamma\sinh\gamma}}\neq 0\right\} \tag{24}$$

and two complex functions z_+ and z_- on U_+ and U_- respectively,

$$z_\pm=(J_1\mp\mathrm{i}J_2)\left(J_q\pm\frac{\sinh\gamma J_3}{\sqrt{\gamma\sinh\gamma}}\right)^{-1}. \tag{25}$$

In $U_+\cap U_-$ we have

$$z_+z_-=1. \tag{26}$$

From the definition of complex coordinates z_+ and z_- intrdocued on M_0^q in (25), we get the expressions of J_i $(i=1,2,3)$ in terms of z_+ and z_-:

$$\begin{aligned}&J_1=-J_q\frac{z_\pm+\bar{z}_\pm}{1+z_\pm\bar{z}_\pm}\qquad J_2=\mp\mathrm{i}J_q\frac{z_\pm-\bar{z}_\pm}{1+z_\pm\bar{z}_\pm}\\&\frac{\sinh\gamma J_3}{\sqrt{\gamma\sinh\gamma}}=\mp J_q\frac{1-z_\pm\bar{z}_\pm}{1+z_\pm\bar{z}_\pm}.\end{aligned} \tag{27}$$

The q-deformed symplectic form (23) becomes

$$\begin{aligned}\omega_q|U_\pm&=2\mathrm{i}J_q\left(J_q^2\gamma^2\frac{(1+z_\pm\bar{z}_\pm)^2}{(1+z_\pm\bar{z}_\pm)^2}+\frac{\gamma}{\sinh\gamma}\right)^{-1/2}\frac{\mathrm{d}\bar{z}_\pm\wedge\mathrm{d}z_\pm}{(1+z_\pm\bar{z}_\pm)^2}\\&=-\mathrm{i}Q_\pm\,\mathrm{d}\bar{z}_\pm\wedge\mathrm{d}z_\pm\end{aligned}$$

where

$$Q_\pm=-2J_q\left(J_q^2\gamma^2\frac{(1-z_\pm\bar{z}_\pm)^2}{(1+z_\pm\bar{z}_\pm)^2}+\frac{\gamma}{\sinh\gamma}\right)^{-1/2}(1+z_\pm\bar{z}_\pm)^{-2}. \tag{29}$$

Since ω_q is closed, it should locally be exact on the open set U_+ and U_-, i.e.

$$\omega_q|U_\pm=\mathrm{d}\theta_\pm. \tag{30}$$

Here the symplectic 1-forms $\theta_\pm$ read

$$\theta_\pm = -\frac{\mathrm{i}}{\gamma z_\pm}\left[\sinh^{-1}\left(J_q\sqrt{\gamma\sinh\gamma}\,\frac{1-z_\pm\bar{z}_\pm}{1+z_\pm\bar{z}_\pm}\right) - \sinh^{-1}(J_q\sqrt{\gamma\sinh\gamma})\right]\mathrm{d}z_\pm$$
$$= -\mathrm{i}p_\pm\,\mathrm{d}z_\pm \tag{31}$$

where

$$p_\pm = \frac{1}{\gamma z_\pm}\left(\sinh^{-1}\left(J_q\sqrt{\gamma\sinh\gamma}\,\frac{1-z_\pm\bar{z}_\pm}{1+z_\pm\bar{z}_\pm}\right) - \sinh^{-1}(J_q\sqrt{\gamma\sinh\gamma})\right). \tag{32}$$

Let us rewrite expressions (27) in terms of variables $z_\pm$ and $p_\pm$,

$$\begin{aligned} J_1 &= -\frac{1}{\sqrt{\gamma\sinh\gamma}}\left[\cosh\left(\frac{\gamma}{2}z_\pm p_\pm\right) z_\pm \sinh\left(\frac{\gamma}{2}(z_\pm p_\pm + 2b)\right)\right. \\ &\qquad \left. -\cosh\left(\frac{\gamma}{2}(z_\pm p_\pm + 2b)\right)\frac{1}{z_\pm}\sinh\left(\frac{\gamma}{2}z_\pm p_\pm\right)\right] \\ J_2 &= \frac{\mp\mathrm{i}}{\sqrt{\gamma\sinh\gamma}}\left[\cosh\left(\frac{\gamma}{2}z_\pm p_\pm\right) z_\pm \sinh\left(\frac{\gamma}{2}(z_\pm p_\pm + 2b)\right)\right. \\ &\qquad \left. +\cosh\left(\frac{\gamma}{2}(z_\pm p_\pm + 2b)\right)\frac{1}{z_\pm}\sinh\left(\frac{\gamma}{2}z_\pm p_\pm\right)\right] \\ J_3 &= \mp(z_\pm p_\pm + b). \end{aligned} \tag{33}$$

Here, for convenience, we have used the relation

$$\sinh\gamma b = J_q\sqrt{\gamma\sinh\gamma}. \tag{34}$$

The Hamiltonian vector fields of z and p are

$$X_{z_\pm} = -\mathrm{i}Q_\pm^{-1}\frac{\partial}{\partial\bar{z}_\pm} \qquad X_{p_\pm} = \mathrm{i}\frac{\partial}{\partial z_\pm} - \mathrm{i}Q_\pm^{-1}\frac{\partial p_\pm}{\partial z_\pm}\frac{\partial}{\partial\bar{z}_\pm} \tag{35}$$

where the relation $\partial p_\pm/\partial\bar{z}_\pm = Q_\pm$ has been used.

Just as for the standard Hamiltonian system (M_0, ω_0, H), the geometric quantization of the deformed Hamiltonian system (M_0^q, ω_q, H_q) is described by the quantization line bundle L_q and the polarization F. In the case under consideration now, such a quantum line bundle L_q exists if and only if $h^{-1}\omega_q$ defines an integral de Rham cohomology class, i.e., the de Rham cohomology class $[-h^{-1}\omega_q]$ of $-h^{-1}\omega_q$ should be integrable. integrating the right-hand side of (23) over the symplectic manifold M_0^q, we have

$$\int_{M_0^q}\omega_q = \frac{1}{J_q^2}\left[2V + \pi\left(J_q^2\frac{\tanh\gamma J_3}{\gamma} - \frac{\gamma J_3 - \tanh\gamma J_3}{\gamma^2\sinh\gamma}\right)\right]\Bigg|_{\frac{\sinh\gamma J_3}{\sqrt{\gamma\sinh\gamma}}=-J_q}^{\frac{\sinh\gamma J_3}{\sqrt{\gamma\sinh\gamma}}=+J_q}. \tag{36}$$

$$V=\int_{M_0^q} \mathrm{d}J_1\,\mathrm{d}J_2\,\mathrm{d}J_3=\left(\pi J_q^2 J_3-\frac{\pi}{2}\,\frac{\sinh 2\gamma J_3-2\gamma J_3}{2\gamma^2\sinh\gamma}\right)\Bigg|_{\frac{\sinh\gamma J_3}{\sqrt{\gamma\sinh\gamma}}=-J_q}^{\frac{\sinh\gamma J_3}{\sqrt{\gamma\sinh\gamma}}=+J_q}. \tag{37}$$

Therefore

$$\int_{M_0^q}\omega_q=-4\pi\frac{\sinh^{-1}(\sqrt{\gamma\sinh\gamma}J_q}{\gamma}. \tag{38}$$

Setting

$$\frac{\sinh^{-1}(\sqrt{\gamma\sinh\gamma}J_q}{\gamma}=j\hbar \tag{39}$$

we get

$$-h^{-1}\int_{M_0^q}\omega_q=-4\pi h^{-1}j\hbar=2j \tag{40}$$

which must be in an integer if $[-h^{-1}\omega_q]$ is integrable. Therefore, $2j\in Z$ and j is an integer or half integer. It is clear now that J_q takes some special values according to j,

$$J_q=\frac{\sinh\gamma j\hbar}{\sqrt{\gamma\sinh\gamma}}. \tag{41}$$

Comparing formulae (34) and (41) we know that here b takes integral and half-integral times of Planck constant.

As for a suitable polarization let us consider the linear frame fields $X_{z_\pm}$,

$$X_{z_\pm}=-\mathrm{i}Q_\pm^{-1}\frac{\partial}{\partial\bar{z}_\pm}. \tag{42}$$

For each $x\in U_+\cap U_-$, we have

$$X_{z_-}=-z_+^{-2}X_{z_\times}.$$

Hence, X_{z_+} and X_{z_-} span a complex distribution F on M_0^q and F is a polarization of symplectic manifold (M_0^q,ω_q). Moreover,

$$\mathrm{i}\omega_q(X_{z_\pm},\bar{X}_{z_\pm})=-\frac{1}{2J_q}\left(J_q^2\gamma^2\frac{(1-z_\pm\bar{z}_\pm)^2}{(1+z_\pm\bar{z}_\pm)^2}+\frac{\gamma}{\sinh\gamma}\right)^{1/2}(1+z_\pm\bar{z}_\pm)^2>0. \tag{43}$$

This means that F is a complete strongly admissible positive polarization of (M_0^q,ω_q).

To get the quantum operator expressions of J_i, $i=1,2,3$, we start from the quantum operators of p and z. For the polarization preserving functions p and z, from (35), we have

$$[X_{p_\pm},X_{z_\pm}]=0. \tag{44}$$

Therefore, their quantum counterparts are

$$\hat{p}_\pm=-\hbar\frac{\partial}{\partial z_\pm}\qquad \hat{z}_\pm=z_\pm \tag{45}$$

where the terms with derivative $\partial/\partial\bar{z}$ have been omitted as the section space is covariantly constant along the polarization F, i.e., the quantum representation space is the holomorphic section space. So the quantum commutator of $\hat{p}$ and $\hat{z}$ is simply

$$[\hat{z}_{\pm}, \hat{p}_{\pm}] = \hbar. \tag{46}$$

having (33) and (45), we obtain the quantum operators with suitable ordering as follows:

$$\begin{aligned}
\hat{J}_1 &= -\frac{1}{\sqrt{\gamma \sinh \gamma}}\left[\cosh\left(\frac{\gamma\hbar}{2} z_{\pm}\frac{\partial}{\partial z_{\pm}}\right) z_{\pm} \sinh\left(\frac{\gamma\hbar}{2}\left(-z_{\pm}\frac{\partial}{\partial z_{\pm}} + 2j\right)\right)\right.\\
&\qquad\left. + \cosh\left(\frac{\gamma\hbar}{2}\left(-z_{\pm}\frac{\partial}{\partial z_{\pm}} + 2j\right)\right)\frac{1}{z_{\pm}}\sinh\left(\frac{\gamma\hbar}{2} z_{\pm}\frac{\partial}{\partial z_{\pm}}\right)\right]\\
\hat{J}_2 &= \frac{\mp \mathrm{i}}{\sqrt{\gamma \sinh \gamma}}\left[\cosh\left(\frac{\gamma\hbar}{2} z_{\pm}\frac{\partial}{\partial z_{\pm}}\right) z_{\pm} \sinh\left(\frac{\gamma\hbar}{2}\left(-z_{\pm}\frac{\partial}{\partial z_{\pm}} + 2_j\right)\right)\right.\\
&\qquad\left. - \cosh\left(\frac{\gamma\hbar}{2}\left(-z_{\pm}\frac{\partial}{\partial z_{\pm}} + 2j\right)\right)\frac{1}{z_{\pm}}\sinh\left(\frac{\gamma\hbar}{2} z_{\pm}\frac{\partial}{\partial z_{\pm}}\right)\right]\\
\hat{J}_3 &= \mp\hbar\left(-z_{\pm}\frac{\partial}{\partial z_{\pm}} + j\right).
\end{aligned} \tag{47}$$

They reduce to (13) when $\gamma \to 0$. Formulae in (47) give the Lie bracket realization of the quantum enveloping algebra $SU_{q,\hbar}(2)$

$$\begin{aligned}
&[\hat{J}_1, \hat{J}_2] = -\frac{\mathrm{i}\sinh \gamma\hbar}{2\gamma}[2\hat{J}_3]_q\\
&[\hat{J}_2, \hat{J}_3] = -\mathrm{i}\hbar\hat{J}_1 \qquad [\hat{J}_3, \hat{J}_1] = -\mathrm{i}\hbar\hat{J}_2
\end{aligned} \tag{48}$$

or

$$[\hat{J}_+, \hat{J}_-] = -\frac{\sinh(\gamma\hbar)}{\gamma}[2\hat{J}_3]_q \qquad [\hat{J}_3, \hat{J}_{\pm}] = \mp\hbar\hat{J}_{\pm}. \tag{49}$$

For the quantum algebra $SU_{q,\hbar}(2)$, Hopf operations: coproduct, counit and antipode can be defined done as usual:

$$\begin{aligned}
&\Delta(\hat{J}_3) = \hat{J}_3 \otimes 1 + 1 \otimes \hat{J}_3\\
&\Delta(\hat{J}_{\pm}) = \hat{J}_{\pm} \otimes \mathrm{e}^{\gamma\hat{J}_3} + \mathrm{e}^{-\gamma\hat{J}_3} \otimes \hat{J}_{\pm}\\
&\eta(\hat{J}_{\pm}) = -\mathrm{e}^{\gamma\hat{J}_3}\hat{J}_{\pm}\,\mathrm{e}^{-\gamma\hat{J}_3} \qquad \eta(\hat{J}_3) = -\hat{J}_3\\
&\varepsilon(\hat{J}_{\pm}) = \varepsilon(\hat{J}_3) = 0 \qquad \varepsilon(1) = 1
\end{aligned} \tag{50}$$

Therefore the quantum group $SU_{q,\hbar}(2)$ is realized in the quantum symmetric top system with non-trivial Hopf algebra structure.

For the quantum q-deformed symmetric top system, using the quantum group $SU_{q,\hbar}(2)$ symmetry, we can write the Heisenberg equations as

$$\begin{aligned}
&\mathrm{i}\hbar\dot{\hat{J}}_1 = [\hat{J}_1, \hat{H}_q] = \frac{I - I_3}{2II_3}(\hat{J}_2\hat{J}_3 + \hat{J}_3\hat{J}_2)\\
&\mathrm{i}\hbar\dot{\hat{J}}_2 = [\hat{J}_2, \hat{H}_q] = \frac{I_3 - I}{2II_3}(\hat{J}_1\hat{J}_3 + \hat{J}_3\hat{J}_1)\\
&\mathrm{i}\hbar\dot{\hat{J}}_3 = [\hat{J}_3, \hat{H}_q] = 0.
\end{aligned} \tag{51}$$

These are the same as those of the quantum undeformed symmetric top system. So the quantum q-deformed symmetric top system and the quantum undeformed symmetric top system correspond to the same Heisenberg equations in quantum mechanics. It is not difficult to check that the Wigner D-functions are still stationary states of the quantum q-deformed symmetric top system and the eigenvalues of the q-deformed Hamiltonian are

$$E^q_{JK} = \frac{1}{2I}[J\hbar]_q[(J+1)\hbar]_q + \frac{I-I_3}{2II_3}\hbar^2K^2 \tag{52}$$

By means of symplectic geometry, we have shown an exactly solved quantum deformed symmetric top model which possesses quantum group symmetry. The energy levels of the model naturally give the nonlinear modifications to the standard quantum symmetric top model. As long as suitable parameters are chosen, the model can give a satisfied description of molecules and nuclei. All these relevant subjects are under investigation [16].

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